Risk Parity & Budgeting with Python

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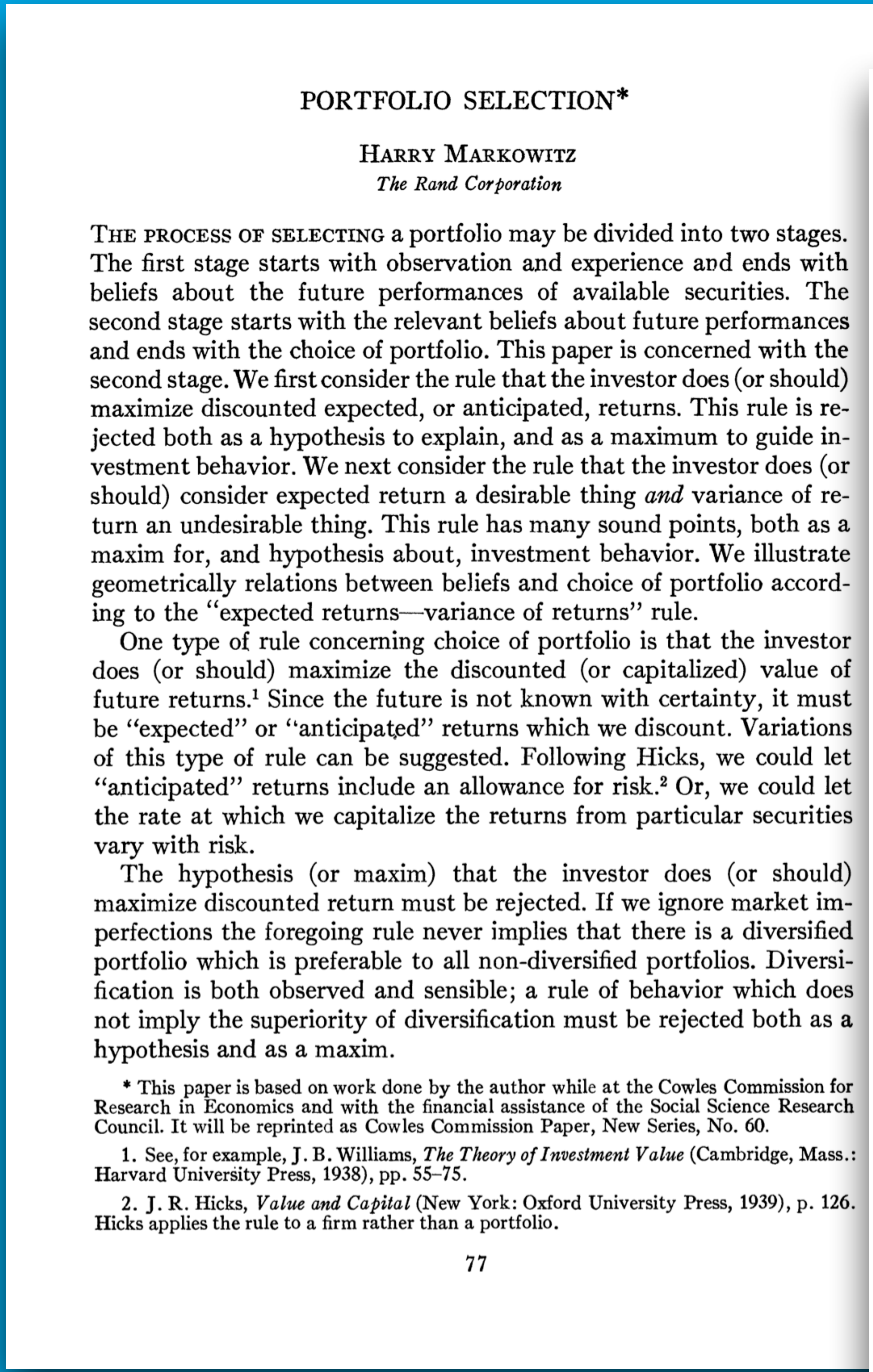
Case Study: Asset Allocation



PORTFOLIO SELECTION*

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THE PROCESS OF SELECTING a portfolio may be divided into two stages. The first stage starts with observation and experience and ends with beliefs about the future performances of available securities. The second stage starts with the relevant beliefs about future performances and ends with the choice of portfolio. This paper is concerned with the second stage. We first consider the rule that the investor does (or should) maximize discounted expected, or anticipated, returns. This rule is rejected both as a hypothesis to explain, and as a maximum to guide investment behavior. We next consider the rule that the investor does (or should) consider expected return a desirable thing *and* variance of return an undesirable thing. This rule has many sound points, both as a maxim for, and hypothesis about, investment behavior. We illustrate geometrically relations between beliefs and choice of portfolio according to the "expected returns—variance of returns" rule.

One type of rule concerning choice of portfolio is that the investor does (or should) maximize the discounted (or capitalized) value of future returns.[1] Since the future is not known with certainty, it must be "expected" or "anticipated" returns which we discount. Variations of this type of rule can be suggested. Following Hicks, we could let "anticipated" returns include an allowance for risk.[2] Or, we could let the rate at which we capitalize the returns from particular securities vary with risk.

The hypothesis (or maxim) that the investor does (or should) maximize discounted return must be rejected. If we ignore market imperfections the foregoing rule never implies that there is a diversified portfolio which is preferable to all non-diversified portfolios. Diversification is both observed and sensible; a rule of behavior which does not imply the superiority of diversification must be rejected both as a hypothesis and as a maxim.

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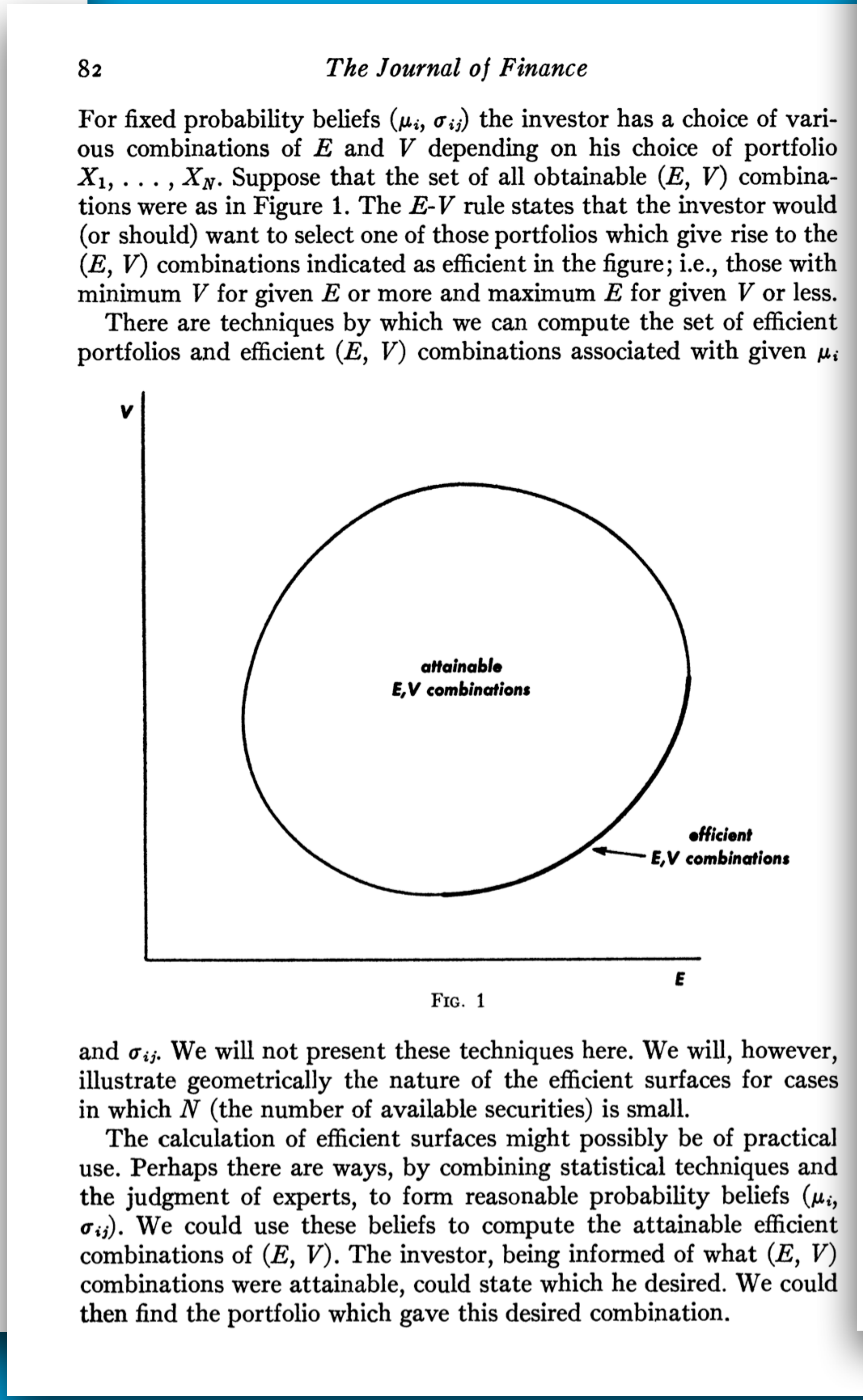
[1] See, for example, J. B. Williams, *The Theory of Investment Value* (Cambridge, Mass.: Harvard University Press, 1938), pp. 55–75.

[2] J. R. Hicks, *Value and Capital* (New York: Oxford University Press, 1939), p. 126. Hicks applies the rule to a firm rather than a portfolio.

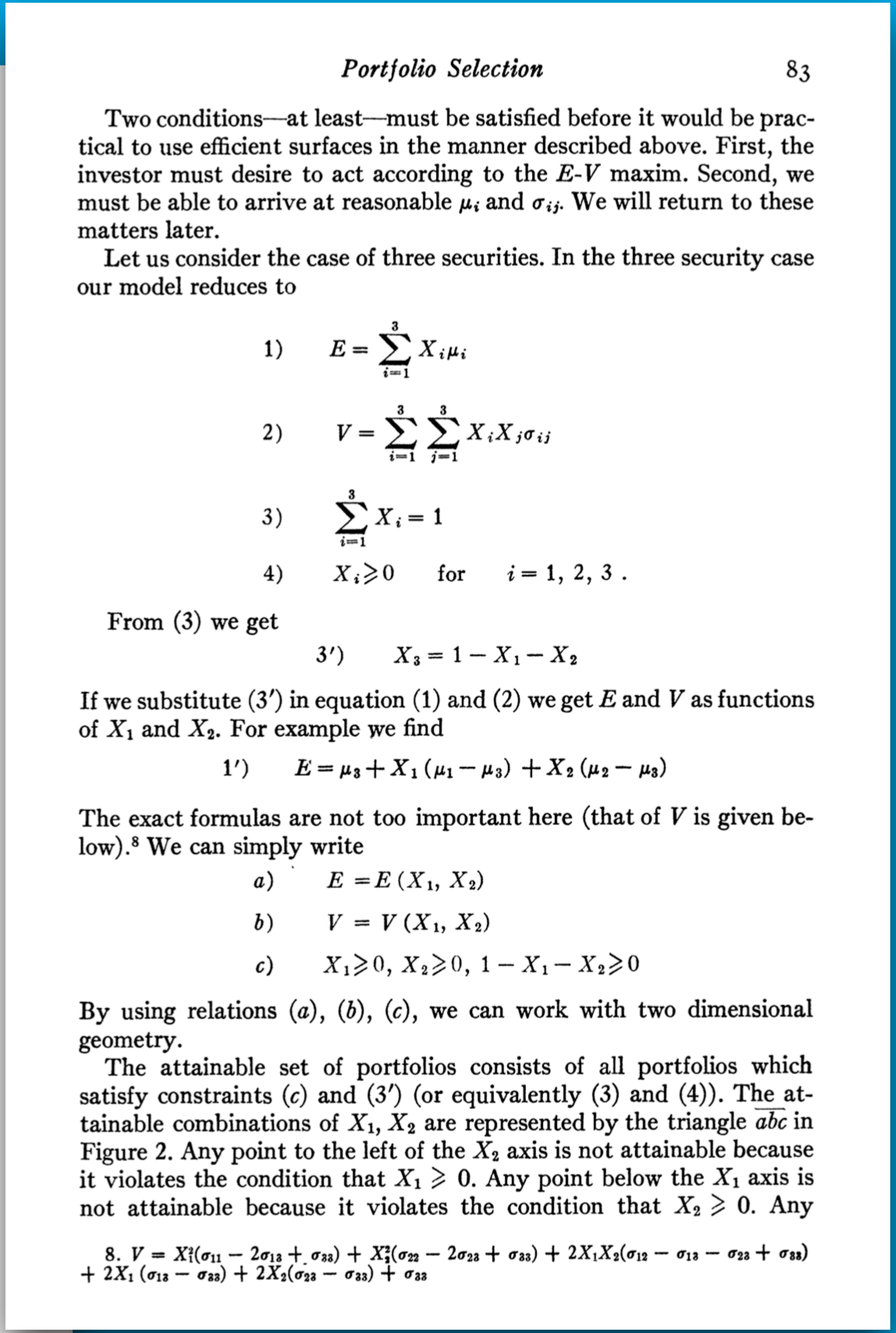


For fixed probability beliefs (μ_i, σ_{ij}) the investor has a choice of various combinations of E and V depending on his choice of portfolio $X_1, \ldots, X_N$. Suppose that the set of all obtainable (E, V) combinations were as in Figure 1. The E-V rule states that the investor would (or should) want to select one of those portfolios which give rise to the (E, V) combinations indicated as efficient in the figure; i.e., those with minimum V for given E or more and maximum E for given V or less.

There are techniques by which we can compute the set of efficient portfolios and efficient (E, V) combinations associated with given μ_i and σ_{ij}. We will not present these techniques here. We will, however, illustrate geometrically the nature of the efficient surfaces for cases in which N (the number of available securities) is small.

FIG. 1

The calculation of efficient surfaces might possibly be of practical use. Perhaps there are ways, by combining statistical techniques and the judgment of experts, to form reasonable probability beliefs (μ_i, σ_{ij}). We could use these beliefs to compute the attainable efficient combinations of (E, V). The investor, being informed of what (E, V) combinations were attainable, could state which he desired. We could then find the portfolio which gave this desired combination.



Two conditions—at least—must be satisfied before it would be practical to use efficient surfaces in the manner described above. First, the investor must desire to act according to the E-V maxim. Second, we must be able to arrive at reasonable μ_i and σ_{ij}. We will return to these matters later.

Let us consider the case of three securities. In the three security case our model reduces to

$$1)\qquad E = \sum_{i=1}^{3} X_i \mu_i$$

$$2)\qquad V = \sum_{i=1}^{3} \sum_{j=1}^{3} X_i X_j \sigma_{ij}$$

$$3)\qquad \sum_{i=1}^{3} X_i = 1$$

$$4)\qquad X_i \geqslant 0 \qquad \text{for} \qquad i = 1, 2, 3\,.$$

From (3) we get

$$3')\qquad X_3 = 1 - X_1 - X_2$$

If we substitute (3′) in equation (1) and (2) we get E and V as functions of X_1 and X_2. For example we find

$$1')\qquad E = \mu_3 + X_1(\mu_1 - \mu_3) + X_2(\mu_2 - \mu_3)$$

The exact formulas are not too important here (that of V is given below).[8] We can simply write

$$a)\qquad E = E(X_1, X_2)$$

$$b)\qquad V = V(X_1, X_2)$$

$$c)\qquad X_1 \geqslant 0,\ X_2 \geqslant 0,\ 1 - X_1 - X_2 \geqslant 0$$

By using relations (*a*), (*b*), (*c*), we can work with two dimensional geometry.

The attainable set of portfolios consists of all portfolios which satisfy constraints (*c*) and (3′) (or equivalently (3) and (4)). The attainable combinations of X_1, X_2 are represented by the triangle $\overline{abc}$ in Figure 2. Any point to the left of the X_2 axis is not attainable because it violates the condition that $X_1 \geqslant 0$. Any point below the X_1 axis is not attainable because it violates the condition that $X_2 \geqslant 0$. Any

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[8] $V = X_1^2(\sigma_{11} - 2\sigma_{13} + \sigma_{33}) + X_2^2(\sigma_{22} - 2\sigma_{23} + \sigma_{33}) + 2X_1X_2(\sigma_{12} - \sigma_{13} - \sigma_{23} + \sigma_{33}) + 2X_1(\sigma_{13} - \sigma_{33}) + 2X_2(\sigma_{23} - \sigma_{33}) + \sigma_{33}$

Primitives

financial assets	$n = 1,2,\ldots,N$
points in time	$t = t_1, t_2, \ldots, t_M$
historical returns	$r_t^n, t = t_1, \ldots, t_M$
mean returns	$\mu_{\mathbf{n}} = \frac{1}{M} \sum_t r_t^n$
expected returns	$\mu = (\mu_1, \mu_2, \ldots, \mu_N)^T$

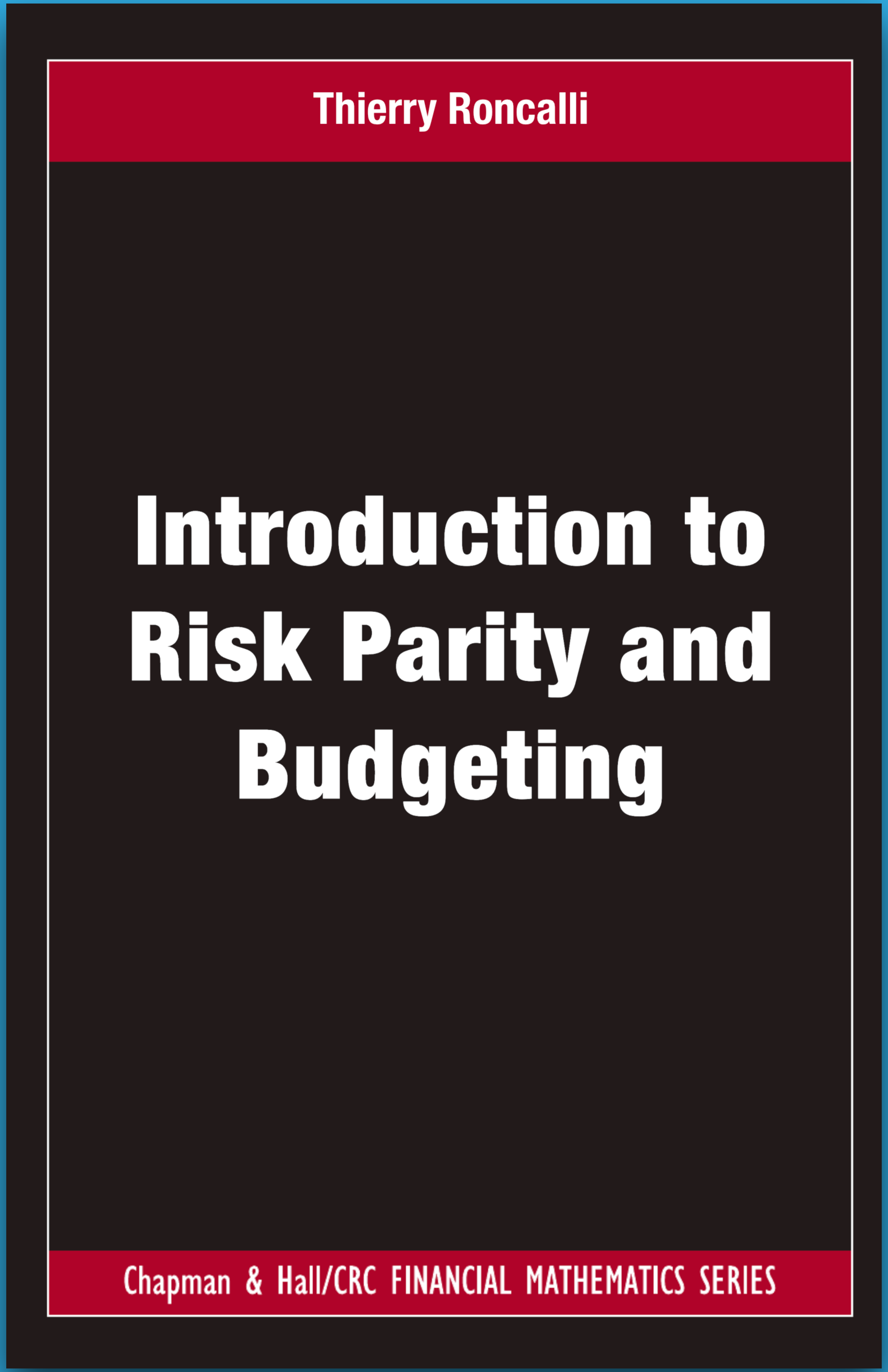
Primitives

variance	$\mathbf{Var}_n = \frac{1}{M} \sum_t \left(r_t^n - \mu_n\right) \left(r_t^n - \mu_n\right)$
volatility (std)	$\sigma = \sqrt{\mathbf{Var}_n}$
covariance	$\mathbf{Cov}_{i,j} = \frac{1}{M} \sum_t \left(r_t^i - \mu_i\right) \left(r_t^j - \mu_j\right)$
covariance matrix	$\boldsymbol{\Sigma} \in \mathbb{R}^{N \times N}$

Portfolio Statistics

weights	$\phi = (\phi_1, \ldots, \phi_N)^T \in \mathbb{R}^N$
portfolio return	$\mu_P = \phi\mu$
portfolio variance	$\mathbf{Var}_P = \phi^T \Sigma \phi$
portfolio volatility	$\sigma_P = \sqrt{\mathbf{Var}_P}$

Case Study: Risk Parity



Thierry Roncalli

Introduction to
Risk Parity and
Budgeting

Chapman & Hall/CRC FINANCIAL MATHEMATICS SERIES

Marginal Risk Contributions

weights	$\phi = (w_1, \ldots, w_N)^T \in \mathbb{R}^N$
risk measure	$\mathscr{R}(\phi)$
risk contribution	$\mathscr{R}\mathscr{C}_n = \phi_n \dfrac{\partial \mathscr{R}(\phi)}{\partial \phi_n}$
Euler decomposition	$\mathscr{R}(\phi) = \sum\limits_n \mathscr{R}\mathscr{C}_n$

Marginal Volatilities

portfolio volatility $\sigma(\phi) = \sqrt{\phi^T \Sigma \phi}$

marginal volatilities $\dfrac{\partial \sigma(\phi)}{\partial \phi} = \dfrac{\Sigma \phi}{\sqrt{\phi^T \Sigma \phi}}$

risk contributions $\mathscr{R}\mathscr{C}_n = \phi_n \cdot \dfrac{(\Sigma \phi)_n}{\sqrt{\phi^T \Sigma \phi}}$

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